$$\mathrm{Nu} = 1.6. \tag{2.8}$$

It can be shown that even when $\sigma \to 0$ the asymptotic relationship (2.8) occurs.

The results of a numerical solution of the problem in the form of c(Re) and Nu(Re) relationships are shown in Fig. 2 by a dashed line for three values of the parameter $\sigma = 0$, 1, and 10. The most characteristic property of the velocity profiles when $\sigma > 0$ is the lack of smoothness in the distribution of velocities through the channel cross section for high Reynolds numbers [Fig. 3, in which for $\sigma = 1$ the w(y) relationships for different Re numbers are shown by solid lines and the temperature distribution θ(Re) by dotted lines]. The Reynolds numbers Re = 1, 10, 100, and 1,000 correspond to values of $a = 2.05$, 0.861, 0.361, and 0.148. The set of w(y) profiles for Re = 1,000 and different Prandtl numbers ($0 < \sigma < 1{,}000$) are shown in Fig. 4. The profiles of the mass velocity $\rho v_x \sim u/\theta$ are smooth (Fig. 5, in which u/θ profiles for $\sigma = 0$ and $\sigma = 1$ are compared at Re = 1,000).

Thus, the unevenness in the density distribution through the channel cross section generates a reduction in the volumetric velocity and an increase in the mass velocity in the area around the axis.

The author wishes to thank A. F. Seleznev for carrying out the calculations and V. N. Shtern for discussing the paper.

LITERATURE CITED

1. B. S. Petukhov, Heating Exchange and Resistance Accompanying the Laminar Flow of a Fluid in Pipes [in Russian], Énergiya, Moscow (1967).

DETERMINING THE RADIUS OF THE AIR VORTEX DURING THE LAMINAR FLOW OF A LIQUID IN A CENTRIFUGAL ATOMIZER

Yu. Z. Nekhamkin, B. D. Strelkov, and Yu. I. Khavkin

UDC 532.517

In existing theories of centrifugal atomizers, such as that of Abramovich [1], in order to determine the radius r_0 of the air vortex the conditions of the maximum rate of flow or some other extremal principle are conventionally employed. In this paper the radius of the air vortex will be determined from the equations of motion of a viscous incompressible liquid.

The atomizer under consideration is illustrated schematically in Fig. 1. Phenomena taking place in the boundary layers close to the ends are not taken into account. The region of flow is divided into two zones.

All the quantities in this paper are dimensionless; lengths are given in terms of the radius of the outlet nozzle r_1, and velocities, in terms of the velocity in the inlet channels V.

In zone I($1 \le r \le a$) the flow is quite flat, of the vortical sink type, i.e., $v = v(r)$, $u = 0$, $w = w(r)$, where v is the radial velocity component, u is the axial component, and w is the circumferential component.

Equations for the velocity components in zone I were obtained in [2]:

$$v = -\varkappa/r;\ w = C_1 r^{1-\varkappa\mathrm{Re}} + C_2/r.$$

where $\varkappa = f/2\pi L r_1$; $\mathrm{Re} = V r_1/\nu$; f is the cross-sectional area of the inlet channels.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 54-57, May-June, 1976. Original article submitted April 8, 1975.

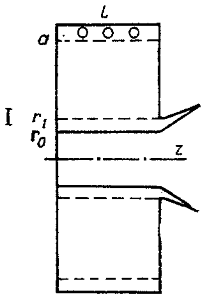

Fig. 1

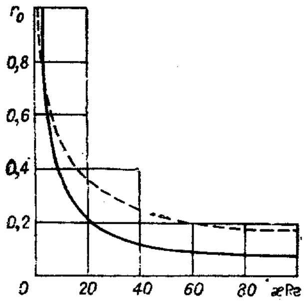

Fig. 2

In order to determine the constants C_1 and C_2 we have the condition

$$w = 1 \ (r = a) \tag{1}$$

and the conjugation conditions for the function f and its derivative at $r=1$.

In zone II ($r_0 \leq r \leq 1$) the liquid is introduced uniformly along the whole length of the chamber L.

For determining r_0 we obtain an integral condition analogous to the integral equation governing the transfer of the principal moment of momentum through the cross section of a laminar jet [3].

Let us consider that even in zone II the circumferential velocity component does not depend on the axial coordinate z.

In zone II we then have the following equations:

$$v\,dw/dr + vw/r = (1/\mathrm{Re})(d^2w/dr^2 + (1/r)dw/dr - w/r^2); \tag{2}$$

$$\partial rv/\partial r + \partial(ru)/\partial z = 0. \tag{3}$$

After multiplying both sides by r and using the continuity equation (3), Eq. (2) may be rewritten in the form

$$\partial(ruw)/\partial z + \partial(rvw)/\partial r + vw = (1/\mathrm{Re})(d/dr)[(1/r)(d(rw)/dr)]$$

or after repeated multiplication by r, in the form

$$\partial(r^2uw)/\partial z + \partial(r^2vw)/\partial r = (1/\mathrm{Re})\{(d/dr)r\,d(rw)/dr - 2d(rw)/dr\}.$$

Integrating both sides of the latter equation across zone II from $r=r_0$ to $r=1$, we obtain

$$\frac{d}{dz}\int_{r_0}^{1} r^2uw\,dr + [r^2vw]_{r=r_0}^{r=1} = \frac{1}{\mathrm{Re}}\left[r\frac{d(rw)}{dr} - 2rw\right]_{r=r_0}^{r=1}. \tag{4}$$

Following [1, 2, 4], we assume that r_0 is constant along the whole length of the chamber.

At $r=r_0$, we have the conditions of impenetrability, and the surface of the air vortex is free, i.e.,

$$v = 0;\ dw/dr = 0 \ (r = r_0), \tag{5}$$

at $r=1$ we have the conjugation condition for the functions v and w.

After substituting boundary conditions (5) into Eq. 4, we obtain

$$\frac{d}{dz}\int_{r_0}^{1} r^2uw\,dr = \varkappa w_1 + \frac{1}{\mathrm{Re}}\left\{r_0w_0 - 2w_1 + \left[\frac{d(rw)}{dr}\right]_{r=1}\right\}, \tag{6}$$

where $w_1 = w(1)$; $w_0 = w(r_0)$.

Equation (6) serves to determine r_0 for known $u(r, z)$ and $w(r)$. For determining the velocity components we make use of approximate expressions, since the solution of the hydrodynamic system of equations involves serious difficulties.

It follows from the condition $r_0 = \mathrm{const}$ that $u = zf(r)$. By equating the amount of liquid passing through the boundary between zones I and II in a section of length z to the flow through the cross section of zone II, we obtain

$$2\pi r_1 v(1) z = 2\pi \int_{r_0}^{1} ru\,dr,$$

from which it follows that u = zf(r).

As u we take the flow-average velocity

$$u = 2\varkappa z/(1 - r_0^2). \tag{7}$$

For w we take the following expression proposed in [4]:

$$w = w_0 2r_0 r/(r^2 + r_0^2).$$

For this specification of w(r) the boundary condition (5) is automatically satisfied. For determining C_1, C_2, and w_0 we have the condition (1) and the two conjugation conditions. Solving this system of equations, we obtain

$$C_1 = \frac{2r_0^2}{2 + 2r_0^2 a^{2-\varkappa \mathrm{Re}} - \varkappa \mathrm{Re}(1 + r_0^2)};\quad C_2 = 1 - C_1 a^{2-\varkappa \mathrm{Re}};$$

$$w_0 = [1 - C_1(1 - a^{2-\varkappa \mathrm{Re}})](1 + r_0^2)/2r_0.$$

The expressions for the circumferential velocity components have the form

$$\begin{gathered} w = C_1 r^{1-\varkappa \mathrm{Re}} + (1 - C_1 a^{2-\varkappa \mathrm{Re}})/r \ (1 \leqslant r \leqslant a); \\ w = r[1 - C_1(1 - a^{2-\varkappa \mathrm{Re}})](1 + r_0^2)/(r^2 + r_0^2) \ (r_0 \leqslant r \leqslant 1). \end{gathered} \tag{8}$$

Substituting the resultant profiles (7) and (8) into the integral equation (6), we obtain an equation linking r_0 to the parameter $\varkappa$Re

$$2\varkappa \mathrm{Re}\, r_0^2 (1 + r_0^2)\left(\frac{1 + r_0^2}{1 - r_0^2} \ln \frac{2r_0^2}{1 + r_0^2} + 1\right) = r_0^4 - 2r_0^2 - 3.$$

Figure 2 illustrates the relationships $r_0 = r_0(\varkappa \mathrm{Re})$ (continuous curve) and $r_0 = (2.513/\varkappa \mathrm{Re}^{1/2})$ (based on the equation of [2], dashed curve).

LITERATURE CITED

1. G. N. Abramovich, "Theory of the centrifugal atomizer," in: Industrial Aerodynamics [in Russian], Izd. TsAGI, Moscow (1944).
2. M. A. Gol'dshtik "Theory of the Rank effect (swirling gas flow in a vortical chamber)," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 1 (1963).
3. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Nauka, Moscow (1973).
4. A. A. Vulis and B. P. Ustimenko, "Aerodynamic flow scheme in a cyclone (dust-catcher) chamber," Vestn. Akad. Nauk KazSSR, No. 4 (1954).